



3.1 A review of sets

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Introduction

- In this topic, we will
 - Define a set
 - Define a subset
 - Define the term disjoint
 - Give examples of each of these





Sets

Definition:

A set is a collection of unique objects.

- For example,
 - The set of all nanotechnology engineering students in 1A
 - The set of all people taller than 3 m
 - The set of all 2-dimensional vectors $\mathbf{R}^2$
 - The set of vectors

$$\left\{ \begin{pmatrix} 2.3 \\ -1.2 \\ 7.8 \end{pmatrix}, \begin{pmatrix} 1.5 \\ 0.9 \\ 6.2 \end{pmatrix} \right\}$$





Sets

- If we are listing the items in a set,
list them between curly braces
$$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5, \mathbf{v}_6\}$$
 - All of these vectors will come from the same vector space
- The set with no objects is denoted with $\{\}$
- *Initially*, however, we will refer to entire vector spaces as a set
 - All the vectors in $\mathbf{R}^6$ are unique and form a set
 - The set of all polynomials in x with real coefficients: $\mathbf{R}[x]$



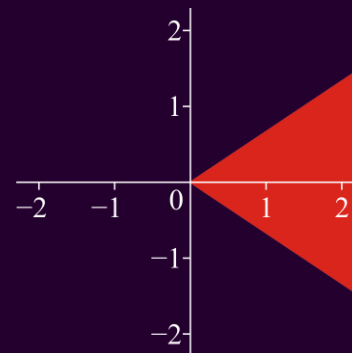
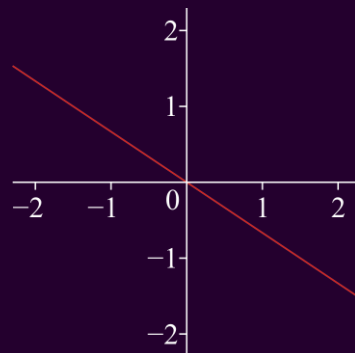


Sets

- Sometimes, we will define sets through a description:
 - The set of all 3-dimensional complex vectors where each entry has an absolute value $|z| < 1$

$$\begin{pmatrix} 0.2 + 0.3j \\ -0.7 - 0.1j \\ 0.5 - 0.5j \end{pmatrix} \quad \begin{pmatrix} 0.6 + 0.8j \\ 0 + 0j \\ 0.2 - 0.9j \end{pmatrix}$$

- Sometimes it will be through an image:





Discrete signals

- I may have a description as follows:
 - The set of all discrete signals that are *bounded*
 - That is, the values never exceed some fixed value in absolute value

$$x = \left(1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots\right)$$

$$y = (0, 10\sin(1), 10\sin(2), 10\sin(3), \dots)$$

$$z = (0, 1, 2, 3, 4, 5, \dots)$$

- Note that the zero vector for discrete signals is bounded:

$$\hat{0} = (0, 0, 0, 0, 0, \dots)$$





Discrete signals

- We may denote a set by an italicized capital Latin letter:
 - The set S of all students at the University of Waterloo
 - The set T of all graduates of secondary schools in Ontario
 - The set U of all complex numbers $|z| \leq 1$
 - The set W of all 3-dimensional vectors with all non-negative entries
- If we are discussing an arbitrary vector space,
we may use a script $\mathcal{U}$ or $\mathcal{V}$





Subsets

- Given two real numbers α and β , the relationship $\alpha \leq \beta$ may or may not be true for two such values
 - The relation tells us how the two values may be related to each other

Definition

One set S is said to be a *subset* of another set T if every element in S is also in T

- This is written as $S \subseteq T$
- Note that for all real numbers $\alpha \leq \beta$, at least one of $\alpha \leq \beta$ or $\beta \leq \alpha$ must be true
- However, it may be that $S \not\subseteq T$ and $T \not\subseteq S$





Subsets

- Examples include:
 - Given the real numbers
 - All positive real numbers
 - All integers
 - All integer multiples of π
 - Given all students at Waterloo
 - All nanotechnology engineering students
 - All students who pass this course
 - Given all students who graduated from Ontario secondary schools
 - All students here are not a subset of this larger set





Subsets

- In this course, we will always be speaking of subsets of a given vector space
 - We may then ask if and how two subsets are related
 - Generally, subsets will be described with a formula
 - All linear combinations of the two vectors

$$\begin{pmatrix} 1.2 \\ 3.7 \\ -5.4 \end{pmatrix} \begin{pmatrix} 6.8 \\ -2.9 \\ -0.1 \end{pmatrix} \qquad \begin{pmatrix} 8.0 \\ 0.8 \\ -5.5 \end{pmatrix} \begin{pmatrix} -5.6 \\ 6.6 \\ -5.3 \end{pmatrix}$$

- All vectors of the form where α is any real value

$$\begin{pmatrix} \alpha + 3 \\ \alpha - 1 \\ \alpha + 1 \end{pmatrix} \qquad \left\{ \begin{pmatrix} 8 \\ 4 \\ 6 \end{pmatrix} \right\}$$





Disjoint subsets

- If two subsets of a given set have no entries in common, we will say that those two subsets are *disjoint*
 - For example,
 - The irrational numbers and the integers are disjoint subsets of the real numbers
 - Given a vector space $\mathcal{V}$, if two subsets both contain the zero vector $\mathbf{0}$, then those subsets are not disjoint





Summary

- Following this topic, you now
 - Understand the definition of a set
 - Understand the definition of a subset and disjoint
 - Are aware of how these will be used in this class





References

- [1] [https://en.wikipedia.org/wiki/Set_\(mathematics\)](https://en.wikipedia.org/wiki/Set_(mathematics))
- [2] <https://en.wikipedia.org/wiki/Subset>





Acknowledgments

Tina Hanna for pointing out an error in a slide.

Mac Qiu for pointing out I used “parentheses” when I meant “braces.”





Colophon

These slides were prepared using the Cambria typeface. Mathematical equations use Times New Roman, and source code is presented using Consolas.

The photographs of flowers and a monarch butter appearing on the title slide and accenting the top of each other slide were taken at the Royal Botanical Gardens in October of 2017 by Douglas Wilhelm Harder. Please see

<https://www.rbg.ca/>

for more information.





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